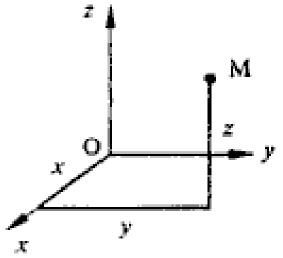
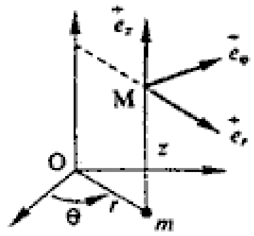
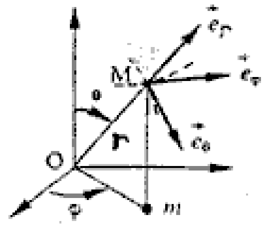
Opérateurs vectoriels

I) Les différents opérateurs

	CARTÉSIENNES	CYLINDRIQUES	SPHÉRIQUES
DÉFINITION	 $V = V(x, y, z)$ $\vec{a} \begin{cases} a_x \\ a_y \\ a_z \end{cases}$	 $V = V(r, \theta, z)$ $\vec{a} \begin{cases} a_r \\ a_\theta \\ a_z \end{cases}$	 $V = V(r, \theta, \varphi)$ $\vec{a} \begin{cases} a_r \\ a_\theta \\ a_\varphi \end{cases}$
GRADIENT	$\vec{\nabla} V \begin{cases} \frac{\partial V}{\partial x} \\ \frac{\partial V}{\partial y} \\ \frac{\partial V}{\partial z} \end{cases}$	$\vec{\nabla} V \begin{cases} \frac{\partial V}{\partial r} \\ \frac{1}{r}\frac{\partial V}{\partial \theta} \\ \frac{\partial V}{\partial z} \end{cases}$	$\vec{\nabla} V \begin{cases} \frac{\partial V}{\partial r} \\ \frac{1}{r}\frac{\partial V}{\partial \theta} \\ \frac{1}{r \sin\theta}\frac{\partial V}{\partial \varphi} \end{cases}$
DIVERGENCE	$\operatorname{div} \vec{a} = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$	$\operatorname{div} \vec{a} = \frac{1}{r}\frac{\partial}{\partial r}(r a_r) + \frac{1}{r}\frac{\partial a_\theta}{\partial \theta} + \frac{\partial a_z}{\partial z}$	$\operatorname{div} \vec{a} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 a_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(a_\theta \sin\theta) + \frac{1}{r \sin\theta}\frac{\partial a_\varphi}{\partial \varphi}$
ROTATIONNEL	$\vec{\text{rot}}\,\vec{a} \begin{cases} \frac{\partial a_z}{\partial y} - \frac{\partial a_y}{\partial z} \\ \frac{\partial a_x}{\partial z} - \frac{\partial a_z}{\partial x} \\ \frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y} \end{cases}$	$\vec{\text{rot}}\,\vec{a} \begin{cases} \frac{1}{r}\frac{\partial a_z}{\partial \theta} - \frac{\partial a_\theta}{\partial z} \\ \frac{\partial a_r}{\partial z} - \frac{\partial a_z}{\partial r} \\ \frac{1}{r}\left[\frac{\partial (r a_\theta)}{\partial r} - \frac{\partial a_r}{\partial \theta}\right] \end{cases}$	$\vec{\text{rot}}\,\vec{a} \begin{cases} \frac{1}{r \sin\theta}\left(\frac{\partial}{\partial \theta}(a_\varphi \sin\theta) - \frac{\partial a_\theta}{\partial \varphi}\right) \\ \frac{1}{r \sin\theta}\frac{\partial a_R}{\partial \varphi} - \frac{1}{r}\frac{\partial}{\partial r}(r a_\varphi) \\ \frac{1}{r}\left[\frac{\partial}{\partial r}(r a_\theta) - \frac{\partial a_r}{\partial \theta}\right] \end{cases}$
LAPLACIEN	$\Delta V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$	$\Delta V = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} + \frac{\partial^2 V}{\partial z^2}$	$\Delta V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \varphi^2}$

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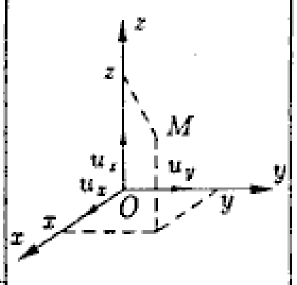
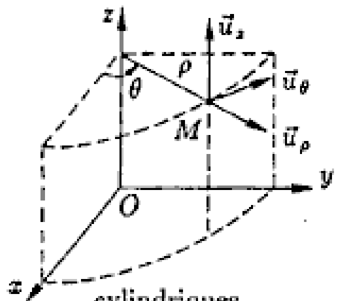
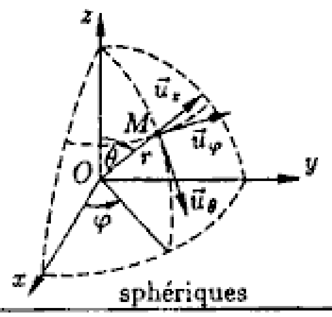
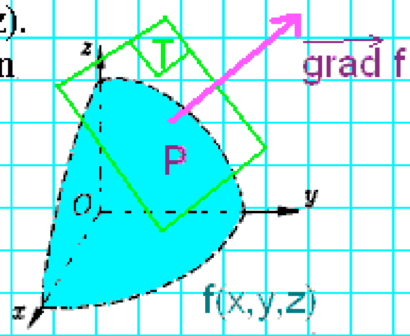
coordonnées	cartésiennes	cylindriques	sphériques
			
$d\vec{l}$	$dx\,\vec{u}_x + dy\,\vec{u}_y + dz\,\vec{u}_z$	$d\rho\,\vec{u}_\rho + \rho d\theta\,\vec{u}_\theta + dz\,\vec{u}_z$	$dr\,\vec{u}_r + r d\theta\,\vec{u}_\theta + r\sin\theta d\varphi\,\vec{u}_\varphi$
$\overrightarrow{\text{grad}}\, f(M)$	$\frac{\partial f}{\partial x}\vec{u}_x + \frac{\partial f}{\partial y}\vec{u}_y + \frac{\partial f}{\partial z}\vec{u}_z$	$\frac{\partial f}{\partial \rho}\vec{u}_\rho + \frac{1}{\rho}\frac{\partial f}{\partial \theta}\vec{u}_\theta + \frac{\partial f}{\partial z}\vec{u}_z$	$\frac{\partial f}{\partial r}\vec{u}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\vec{u}_\theta + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\vec{u}_\varphi$
$\text{div}\,\vec{U}(M)$	$\frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} + \frac{\partial U_z}{\partial z}$	$\frac{1}{\rho}\left(\frac{\partial(\rho U_\rho)}{\partial \rho} + \frac{\partial U_\theta}{\partial \theta}\right) + \frac{\partial U_z}{\partial z}$	$\frac{1}{r^2}\frac{\partial(r^2 U_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(U_\theta \sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial U_\varphi}{\partial \varphi}$
$\overrightarrow{\text{rot}}\, U(M)$	$\left[\frac{\partial U_z}{\partial y} - \frac{\partial U_y}{\partial z}\right]\vec{u}_x +$ $\left[\frac{\partial U_x}{\partial z} - \frac{\partial U_z}{\partial x}\right]\vec{u}_y +$ $\left[\frac{\partial U_y}{\partial x} - \frac{\partial U_x}{\partial y}\right]\vec{u}_z +$	$\left[\frac{1}{\rho}\frac{\partial U_z}{\partial \theta} - \frac{\partial U_\theta}{\partial z}\right]\vec{u}_\rho +$ $\left[\frac{\partial U_\rho}{\partial z} - \frac{\partial U_z}{\partial \rho}\right]\vec{u}_\theta +$ $\left[\frac{1}{\rho}\frac{\partial(\rho U_\theta)}{\partial \rho} - \frac{1}{\rho}\frac{\partial U_\rho}{\partial \theta}\right]\vec{u}_z$	$\frac{1}{r\sin\theta}\left[\frac{\partial(\sin\theta\, U_\varphi)}{\partial \theta} - \frac{\partial U_\theta}{\partial \varphi}\right]\vec{u}_r +$ $\left[\frac{1}{r\sin\theta}\frac{\partial U_r}{\partial \varphi} - \frac{1}{r}\frac{\partial(r U_\varphi)}{\partial r}\right]\vec{u}_\theta +$ $\frac{1}{r}\left[\frac{\partial(r U_\theta)}{\partial r} - \frac{\partial U_r}{\partial \theta}\right]\vec{u}_\varphi$
$\Delta f(M)$	$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) +$ $\frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \varphi^2}$

II) Interprétation géométrique des opérateurs vectoriels

1) Le gradient

Soit une surface définie par une fonction f(x,y,z).
Le gradient de f définit le vecteur normal au plan tangent en tout point de cette surface.



2) Le rotationnel

Le rotationnel "définit" le centre de rotation instantané d'un champs de vecteurs.

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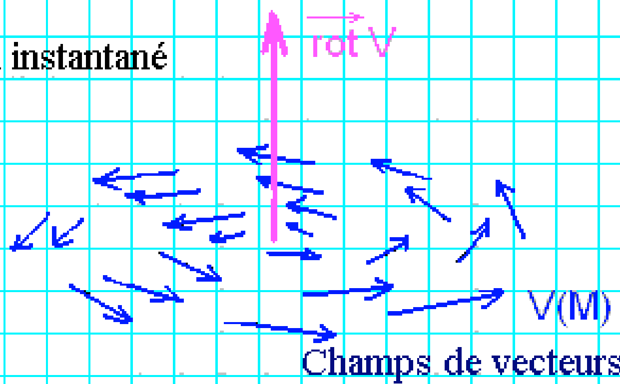

III) Relations entre opérateurs vectoriels

$$\mathrm{div}\,\vec{V} = \vec{\nabla}\,.\,\vec{V}$$

$$\overrightarrow{\mathrm{rot}\,\vec{V}} = \vec{\nabla} \wedge \vec{V}$$

IV) Intervention des opérateurs vectoriels dans les intégrales

$$\iint \vec{E}\,.\,\vec{n}\;\;dS \;= \iiint \mathrm{div}\,\vec{E}\;dv$$ Théorème de Gauss

$$\int \vec{A}\,.\,\vec{dl} \;= \iint \overrightarrow{\mathrm{rot}\,\vec{A}}\,.\,\vec{n}\;\;dS$$ Théorème de Stokes